Cambridge Pre-U

Cambridge Assessment International Education
Cambridge Pre-U Certificate

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers..

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

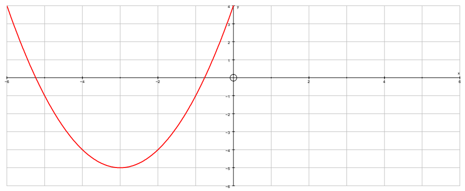
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Question	Answer	Marks	Guidance
1(a)	$x^2 + 6x + 4 = (x+3)^2 - 5$	**B1**	
1(b)		**B1**	$(-3, -5)$ **FT** stated or seen on graph (or **CAO** from calculus work)
		B1	Apparent parabola, vertex in correct quadrant, intersecting the y-axis visibly at $(0, 4)$, and crossing x-axis twice

Question	Answer	Marks	Guidance
2(a)	$u_n = a + (n-1)d$	**M1**	Correct formula used with $a = 100$ and $d = \pm 2.5$
	$100 + (n-1)(-2.5) > 0$	**M1**	Form an inequality with $d = -2.5$ used **OR** evaluating **both** $u_{40} = 2.5$ **and** $u_{41} = 0$
	$n < 41$ so $n = 40$	**A1**	40 must be clearly given as final answer Give M1 M0 A1 for correct answer from no/fudged inequality
2(b)	$S_n = \frac{n}{2}(2a + (n-1)d)$	**M1**	Correct formula used with $n = 40$ or 41 and $d = \pm 2.5$ **OR FT** their positive integer n
	2050	**A1**	**CAO**

Question	Answer	Marks	Guidance
3(a)	Area = $\frac{1}{2}(7)(12)\sin 120$	**M1**	Correct *Area* formula used
	Area = $21\sqrt{3}$ or **awrt** 36.4 (cm^2)	**A1**	
3(b)	$AC^2 = 7^2 + 12^2 - 2(7)(12)\cos 120$	**M1**	Correct *Cosine Rule* formula used
	$AC = \sqrt{277}$ or **awrt** 16.6 (cm)	**A1**	

Question	Answer	Marks	Guidance
4(a)	$g(x) = x^4 - 2$	**B1**	
4(b)	$\frac{dy}{dx} = 2\left(\frac{1}{2}x - 1\right)^3$	**B1**	
	At $x = 4$ gradient = 2	**M1**	Substitute $x = 4$ into their $\frac{dy}{dx}$
	$y + 1 = 2(x - 4)$ **or** $y = 2x - 9$	**A1**	**CAO**

Question	Answer	Marks	Guidance
5	$u = x,\ \dfrac{du}{dx} = 1$ and $\dfrac{dv}{dx} = e^{-x},\ v = -e^{-x}$	**M1**	± Correctly chosen parts (incl. using fn. notation) **SOI**
	$\int u\dfrac{dv}{dx} = uv - \int v\dfrac{du}{dx}$	**M1**	Clear use of the correct formula or 'FIS – IDFIS' etc.
	$\int xe^{-x}dx = [-xe^{-x}]_0^1 + \int_0^1 e^{-x}dx = [-xe^{-x}]_0^1 - \left[e^{-x}\right]_0^1$	**A1**	Completely correct integration. Ignore limits
	$\left(-e^{-1} - e^{-1}\right) - (-0-1)$	**M1**	Use of limits in correct order
	$-2e^{-1} + 1$ so $\dfrac{e-2}{e}$	**A1**	**AG** legitimately shown in full

Question	Answer	Marks	Guidance
6(a)	$3 - i$	**B1**	
6(b)	$(z-(3+i))(z-(3-i))(z-2)$	**M1**	Method for finding the (real) quadratic factor
	$(z^2 - 6z + 10)(z-2)$	**A1**	Correct quadratic obtained (all real coeffts.)
	$(z^3 - 8z^2 + 22z - 20) \quad \Rightarrow \quad a = -8,\ b = 22$	**M1 A1**	Full expansion attempted; correct values clearly stated
	Alt. I		
	$z = 3 + i,\ \ z^2 = 8 + 6i,\ \ z^3 = 18 + 26i$ substd. into $z^3 + az^2 + bz - 20$	**M1**	
	gives $(8a + 3b - 2) + (6a + b + 26)i$	**A1**	
	Solving $8a + 3b = 2$ and $6a + b = -26$ simultaneously $\Rightarrow$ $a = -8,\ b = 22$	**M1 A1**	Re & Im parts only need to be identified at 2nd M stage

Question	Answer	Marks	Guidance
6(b)	**Alt. II**		
	$(z-2)(z^2+[a+2]z+10)$ e.g.	**M1 A1**	
	Using sum of roots: $-(a+2)=3+i+3-i \;\Rightarrow\; a=-8 \;\Rightarrow\; b=22$	**M1 A1**	
	Alt. III		
	$\mathrm{f}(2)=0 \;\Rightarrow\; 4a+2b=12$ and $\Sigma\alpha=-a=-8$ then $b=22$		Must be an attempt at both a, b to earn **M**s

Question	Answer	Marks	Guidance
7	$\mathrm{f}(x+h)-\mathrm{f}(x)=\dfrac{1}{x+h}-\dfrac{1}{x}$	**M1**	Numerator with common denominator attempted
	$\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}=\dfrac{1}{h}\left(\dfrac{-h}{x(x+h)}\right)$	**M1**	Attempt at a full expression involving h's or δx's
	$\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}=\dfrac{-1}{x(x+h)}$	**A1**	Obtaining exactly this result (cancelled h may be implicit)
	$\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$	**B1**	Full definition must be seen with use of $h\to 0$ or $h=0$
	$\mathrm{f}'(x)=\dfrac{-1}{x^2}$	**A1**	**CSO** answer obtained from limiting process (only B0 allowed)

Question	Answer	Marks	Guidance
8	$\log_3\left(x^2-3x-10\right)=\log_3 3+\log_3\left(x+2\right)$	**B1**	Seeing $\frac{1}{2}\log_3 9 = \log_3 3$ (or 1) anywhere
	$\log_3(x^2-3x-10)=\log_3 3(x+2)$ **or** $\log_3\left(x^2-3x-10\right)-\log_3\left(x+2\right) = \log_3\dfrac{\left(x^2-3x-10\right)}{\left(x+2\right)}$	**M1**	Applying division or product law of logs to their expression.
	$x^2-3x-10=3(x+2)$ or $\dfrac{(x+2)(x-5)}{(x+2)}=3$	**M1**	Correctly removing the logs
	$x^2-6x-16=0$ or $x-5=3$	**M1**	Solving an equation having no logs present
	$x=8$	**A1**	$x = 8$ *and* −2 earns A0 (No need to justify that $x \neq -2$.)

Question	Answer	Marks	Guidance
9(a)	$(1+2x)^p = 1+2px+\dfrac{p(p-1)(2x)^2}{2!}+\dfrac{p(p-1)(p-2)(2x)^3}{3!}+\ldots$	**M1**	Attempt binomial expansion with at least the x and x^3 terms seen Must have binml. coeffts. and $(2x)^r$ or $2x^r$ (condone ${}^pC_r$)
	$2(\dfrac{p(p-1)(p-2)8}{6}) = 2p$	**M1**	Set Cft.(x) = 2×Cft.(x^3) and attempt to simplify to obtain a cubic eqn. or a quadratic (if $p = 0$ implicitly removed)
	$8p^3 - 24p^2 + 10p = 0$ or $4p^2 - 12p + 5 = 0$	**A1**	Obtain correct simplified cubic/quadratic eqn.
	$p(2p-1)(2p-5) = 0$	**M1**	Method for solving their cubic/quadratic eqn.
	$p = (0), \frac{1}{2}, \frac{5}{2}$	**A1**	**CSO**
	Reject $p = \frac{5}{2}$ since Cft.(x^2) = 7.5 or from $(0 <)\ p < 1$	**B1**	Or at least Cft.(x^2) shown positive
	$p = \frac{1}{2}$	**A1**	**CAO** Condone *not* confirming Cft.(x^2) < 0
9(b)	$\lvert x \rvert < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$	**B1**	

Question	Answer	Marks	Guidance
10(a)	$\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\2\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}0\\1\\1.5\end{pmatrix}+\mu\begin{pmatrix}4\\-1\\1\end{pmatrix}$	**M1**	Lines parametrised (condone equal scalar parameters and/or mixed p.v.s and d.v.s here)
	$(x=)\ 1+2\lambda=4\mu$ $(y=)\ 2+\lambda=1-\mu$ $(z=)\ 3+2\lambda=1.5+\mu$	**M1**	$\geqslant$ 2 expressions equated (parameters MUST be different here)
	$\lambda=-\frac{5}{6}$ and $\mu=-\frac{1}{6}$	**M1 A1**	Two eqns. solved simultaneously; **CAO** value(s) for λ, μ or both
	Checking $\lambda=-\frac{5}{6}$ and $\mu=-\frac{1}{6}$ in their 3rd eqn.	**B1**	Must be visible and correct
	Point of intersection is $\left(-\frac{2}{3},\ \frac{7}{6},\ \frac{4}{3}\right)$	**A1**	**CAO** Allow vector form
10(b)	$\cos\theta=\dfrac{\text{scalar product}}{\text{product of moduli}}$	**M1**	Use of correct formula using *their* direction vectors
	scalar product = 9 moduli are 3 and $\sqrt{18}$ or $3\sqrt{2}$ or 4.24…	**B1**	All correct or **FT** *their* d.v.s (Note: product of moduli is 12.7…)
	$\theta=45°$ or $\frac{1}{4}\pi$	**A1**	**CSO**

Question	Answer	Marks	Guidance
11(a)	$\tan 15° = \tan(60° - 45°)$ or $\tan 15° = \tan(45° - 30°)$	**M1**	Method for finding tan15°. Also $\tan 30° = \tan(2 \times 15°)$.
	$\dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \dfrac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$	**M1**	Correct application of $\tan(A \pm B)$ formula (or double ∠-formula)
	$\tan 60° = \sqrt{3}$, $\tan 45° = 1$, $\tan 30° = \dfrac{1}{\sqrt{3}}$	**M1**	Correct substitution of relevant exact trig. value(s) [For those using $\tan 30° = \tan(2 \times 15°)$ this is for creating a quadratic in t : $t^2 + 2\sqrt{3}t - 1 = 0$]
	$\dfrac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$	**M1**	Rationalisation of denominator [or for finding $t = \tan 15°$ using the quadratic formula]
	$2 - \sqrt{3}$	**A1**	**AG** Nothing incorrect seen. [In $t = \tan 15°$ case, chosen root must be justified.]
	Special Case Max. 3/5		
	Using $\tan 15° = \sin 15° / \cos 15°$ with calculator-derived exact values	**M1**	
	Rationalising denominator of e.g. $\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$ **M1** $= 2 - \sqrt{3}$ **A1 AG**	**M1** **A1 AG**	

Question	Answer	Marks	Guidance
11(b)	$(\cos x \cos\frac{1}{6}\pi - \sin x \sin\frac{1}{6}\pi)(\cos x \cos\frac{1}{6}\pi + \sin x \sin\frac{1}{6}\pi) = \cos 2x$	**B1**	Use of $\cos(A \pm B)$ formulae **BOD** in either order
	$\cos\frac{1}{6}\pi = \frac{1}{2}\sqrt{3}$ and $\sin\frac{1}{6}\pi = \frac{1}{2}$	**B1**	Correct exact values used
	LHS = $\frac{3}{4}\cos^2 x - \frac{1}{4}\sin^2 x$	**M1**	Obtaining LHS in form $a\cos^2 x + b\sin^2 x$ (a, b numerical)
	Use of $c^2 + s^2 = 1$ to obtain c^2 only or s^2 only on LHS	**M1**	**Or** given later when $A\,c^2 = B\,s^2$ (probably leading to t^2)
	Use of $\cos 2x = 2c^2 - 1$ or $1 - 2s^2$ on RHS	**M1**	Matching result used (e.g. $\cos 2x = c^2 - s^2$ if t^2 used later)
	$c^2 = \frac{3}{4}$, $s^2 = \frac{1}{4}$ or $t^2 = \frac{1}{3}$	**A1**	Correct equation obtained for one of c, s or t
	$x = \frac{1}{6}\pi$, $\frac{5}{6}\pi$ (or 0.524, 2.62 or better)	**A1 A1**	**CSO** (ignore extra solutions out-of-range)

Question	Answer	Marks	Guidance
12(a)	$\int \frac{12000}{P(12000-P)}\mathrm{d}P = \int k\mathrm{d}t$	M1	Separating variables ($\pm\int$ signs). Denomr. need not be factorised.
	$\frac{A}{P}+\frac{B}{12000-P}=\frac{12000}{P(12000-P)} \Rightarrow A=1,\ \ B=1$	M1	Partial fraction form (denomr. must be factorised) on LHS
		A1 A1	Check their A, B from placement of the 12000 *and* the k
	$\int(\frac{1}{P}+\frac{1}{12000-P})\mathrm{d}P = k\int \mathrm{d}t \Rightarrow \ln P - \ln(12000-P) = kt + C$	B1	Correctly integrated $\frac{A}{P}+\frac{B}{12000-P}$ terms. Ignore mod. signs.
	$\ln\frac{P}{12000-P} = kt + C$	M1	Use of log law to combine these two log integrals
	$\frac{P}{12000-P} = \mathrm{e}^{kt+C} = A\mathrm{e}^{kt}$	M1 dep.	Removing the logs. Allow e^{kt+C} or e^{kt} at this stage.
			No $+C$ means, generally, A0, M0, A0, M0, A0 following
	$t=0$, $P=500$ used to obtain $A=\frac{1}{23}$ or $C=\ln(\frac{1}{23})=-3.1355\ldots$	A1	From correct working
	$t=3$, $P=2000$ used to calculate k: $\frac{1}{5}=\frac{1}{23}\mathrm{e}^{3k}$ or $\frac{1}{5}=\mathrm{e}^{3k-3.1355\ldots}$	M1	Method to find k from any correct answer form
	$k=0.509$ (NB k **can** be found correctly from the given answer with $t=3$, $P=2000$)	A1	**AG** $k=0.509$ (3 d.p.) correctly found (This M1 A1 can thus be recouped from this 5-mark section)
	$\frac{P}{12000-P}=\frac{1}{23}\mathrm{e}^{0.509t}$ or $\frac{23P}{(12000-P)}=\mathrm{e}^{0.509t}$	M1	Re-writing to make P the subject (correct form)
	$P=\frac{12000\mathrm{e}^{0.509t}}{23+\mathrm{e}^{0.509t}}=\frac{12000}{23\mathrm{e}^{-0.509t}+1}$	A1	**AG** fully justified

Question	Answer	Marks	Guidance
12(b)	12 000	**B1**	
	$e^{-0.509t} \to 0$ as $t \to \infty$	**B1**	Or words to this effect. Calculus arguments score B0. $e^{-kt} = 0$ or ≈ 0 or denomr. $\to 1$ score B0